Tracking Control of Autonomous Car with Attention to Obstacle Using Model Predictive Control

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***Abstract*— Previous research of Model Predictive Control (MPC) focused on the effect of cost function weights to its performance on path tracking and obstacle avoidance. The best performance was obtained when the error weight is greater than the input weight. However, the car movement was still oscillating and avoiding maneuver was still ineffective. Different from the previous research, this paper focuses on finding the best performance by varying the combination of MPC parameters while maintaining the cost function weight ratio following the previous research. This research uses Linear Time Variant MPC (LTV MPC).The trajectory tracking problem is defined by using a time-varying reference. MPC parameters combinations are varied to find the best performing design. In the obstacle avoidance system, obstacle detection is done by measuring the distance between the instant car position and the obstacle position. While an obstacle is detected, a new lateral position constraint is calculated. Trajectory tracking test are done using 2 types of tracks: sine wave and lane changing. Obstacle avoidance tests are done using 1 obstacle and 2 obstacles. Results are evaluated using Root Mean Square Error (RMSE) of car position, cost function, and the nearest distance between car and obstacle. Results show that MPC was able to evade obstacles while tracking the time-varying reference with 0.4 s delay. However, some variations were unable to meet the safe zone constraints for obstacle avoidance.**

Keywords—Autonomous Car, Model Predictive Control, Obstacle Avoidance, Trajectory Tracking.

I. Introduction

Development of control for autonomous vehicles has grown rapidly. The first autonomous car was a radio-controlled car called the “Linriccan Wonder”, developed in 1926 [1]. While in this 21st century, self-driving features are already implemented into several cars. Features includes Cruise Control and Active Lane Assist [2]. However, features only acts as driving aid; it does not allow the car to run without a driver. To realize a fully unmanned autonomous car, a reliable autonomous system is needed.

A fully autonomous car, as explained in SAE J306, is a car that can run in any condition without driver’s intervention. By way of explanation, the Driver Assist System (DAS) handles all the Dynamic Driving Task (DDT) [3]. 2 tasks of DDT are lateral and longitudinal control as well as response to objects and other events. Much research has been done to develop das.

One particular research is the use of Model Predictive Control (MPC) for path tracking and obstacle avoidance. [4] proved MPC can control the car to follow the path while evading obstacles. The research focused on the effect of cost function weights to its performance on path tracking and obstacle avoidance. The best performance was obtained when the error weight is greater than the input weight. However, the car movement was still oscillating and avoiding maneuver was still ineffective.

To overcome the previous research’ problem, the writers propose to use a Linear Time Variant MPC (LTV MPC) for path tracking and obstacle avoidance. In this paper, cost function weight ratio is maintained following [4]. Instead, in this paper the combination MPC parameters, namely the Prediction Horizon (Np) and Control Horizon (Nc), are varied to find the best performing design.

II. Methods

A. Related Works

Current research on autonomous car control includes the usage of MPC as its controller. One of them is the research of MPC ability to handle path tracking and obstacle avoidance problem as done in [4]. In the research, it was found that when the error weight is greater than the input weight in cost function formulation, it produces the least oscillating car movement. It was also proven that MPC can handle initial position error to the desired path. Obstacle avoidance tests were done by placing 3 static obstacles along the reference path. Results showed the car was able to follow the path while evading obstacles. Even though evading maneuver seemed to be ineffective from the fact that it moves farther away from what is needed to avoid the obstacle.

B. Literature Review

1) Kinematic Car Model

One of the well-known models used to describe vehicle kinematics is called the Bicycle Model. In Fig.1, the vehicle has 2 wheels, the front and the back. The back wheel is attached to the vehicle body and the front wheel can rotate about the vertical axis of vehicle to turn it. the vehicle’s velocity in global coordinate is $(v\cos\theta,\ v\sin\theta)$. The kinematic equation is defined by

$$\begin{aligned}\dot{x} &= V\ cos(\theta)\\ \dot{y} &= V\sin(\theta)\\ \dot{\theta} &= \frac{V}{L}\tan(\gamma)\end{aligned} \qquad (1)$$

with L as car length, V as the velocity, and θ as the heading angle. Hence, $\dot{\theta}$ is the turning rate [5].

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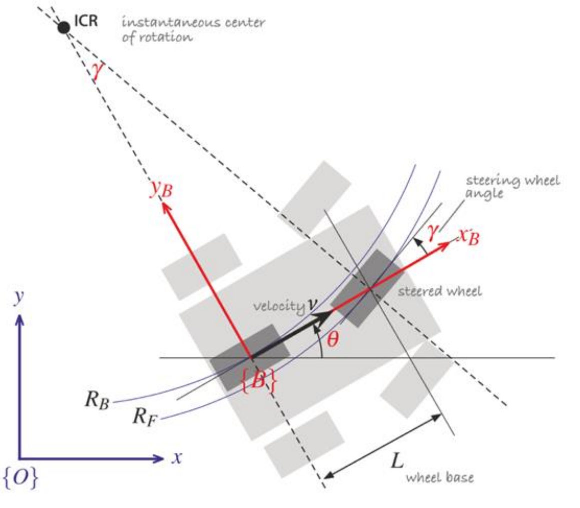

Fig. 1. Bicycle Model

2) Model Predictive Controller

The main concept of MPC is it calculates the system's future input value as a solution to an optimization problem. MPC predicts the system's future output within the determined time horizon [6].

The cost function on time step k is defined by

$$J_k = \sum_{i=0}^{Np} \boldsymbol{x}_{k+i+1}^T Q \boldsymbol{x}_{k+i+1} + \boldsymbol{u}_{k+i}^T R \boldsymbol{u}_{k+i} \quad (2)$$

where Q and R are cost function weights for error and input, respectively, i is the prediction step from time step k, and Np is the prediction horizon [7].

3) Adaptive MPC

Adaptive MPC is an MPC that uses a time-varying internal model. This model is suitable for a system that has a different dynamic according to its operating points [8]. One adaptive MPC method is Linear Time Variant MPC (LTV MPC).

LTV MPC is mainly used to overcome nonlinear systems. In this method, model linearization is done in every time step. Therefore, a new and more accurate linear model is produced according to the new operating point [9].

4) Quadratic Programming

Quadratic programming (QP) is an algorithm developed to find an optimum value (x) that can minimize cost function (J). Generally, cost function is defined by

$$J = \frac{1}{2} x^T E x + x^T F \quad (3)$$

subjected to a constraint defined by

$$Mx \leq \gamma \quad (4)$$

Where E and F are vectors, M is a matrix reflecting constraints and γ is the constraint value vector compatible with the quadratic programming problem.

To get the optimum value, the partial derivative of J to x needs to be 0. Therefore, the optimum x value can be formulated as

$$x = -E^{-1}(M^T \lambda) \quad (5)$$

where λ is called the Lagrange Multiplier, the variable varied to obtain the optimum x value [10].

5) Hildreth's Algorithm

The Hildreth's Algorithm is one of quadratic programming algorithm. This algorithm defines a set of constraints at each time step to be treated as the active set. The set is a subset of constraints that are active on the current point [10]. Hildreth's Algorithm is as written in [11].

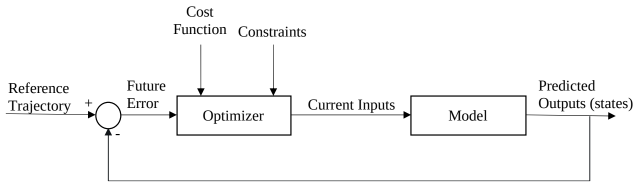

Fig. 2. Model Predictive Control Algorithm [7]

6) Taylor Series Linearization

Linearization can be done using the first-order Taylor series. If there exists a function with a single variable called $f(x)$. Then the linearization of $f(x)$ on the equilibrium point a, where $f(a) = 0$ is fulfilled can be written by

$$f(x) = f(a) + f'(a)(x - a) \quad (6)$$

[12].

While for multiple variable function is defined by $f(x_1, x_2)$, the linearization on the equilibrium point of a and be can be written by

$$f(x_1, x_2) = f(a, b) + [f'(a)(x - a) + f'(b)(x - b)] \quad (7)$$

[13].

C. System Design

1) Car Plant and MPC Internal Plant

The car kinematic model (1) is used for the plant while the linearized kinematic model of (1) is used for the MPC internal model. Linearization was done using the first order Taylor series, the resulting equation can be written in a state space representation as

$$\begin{aligned} \dot{X} &= AX + BU \\ \dot{Y} &= CX \end{aligned} \quad (8)$$

with

$$A = \begin{bmatrix} 0 & 0 & -V\sin(\theta) & 0 \\ 0 & 0 & V\cos(\theta) & 0 \\ 0 & 0 & 0 & \frac{V}{L}\tan^2(\gamma) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ \frac{\tan(\gamma)}{L} & 0 \\ 0 & 1 \end{bmatrix} \quad C = I_4 \quad (9)$$

in accordance to the global coordinate, the states are x position, y position, heading angle, and steering angle $[x \quad y \quad \theta \quad \gamma]^T$. The inputs are car velocity and steering rates, $[V \quad \omega]^T$. The system is then discretized using Zero-Order-Hold (ZOH) method with 0.1 s sampling time.

2) Lanes and Obstacle

The reference point of the ego car is in the center of the car. the car is driving on a 3-lane road along the x-axis. Lane width is 4 m. the lane is defined as y position constraint

$$y_{min} \leq y \leq y_{max} \quad (10)$$

Obstacle avoidance tests were done using a simple dynamic obstacle, meaning the obstacle is a non-moving and non-permanent object placed on the road. obstacle position is predetermined, and object detection is done by measuring the distance between the car and the obstacle. If the distance is less than 50 m, then an object is considered detected. When

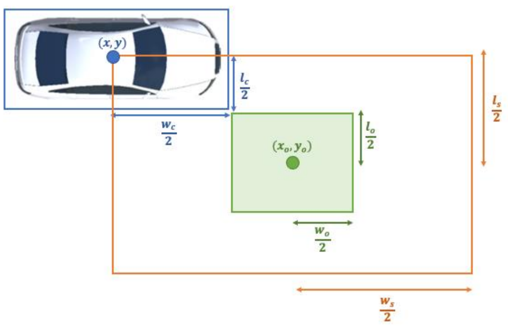

Fig. 3. Safe Zone

an object is detected a new x,y position constraint is defined following the safe zone in Fig.3.

3) Controller Design

The predicted output of MPC is written as

$$\vec{Y} = Fx(k) + \Phi\vec{U}(k) \tag{11}$$

where Np is the prediction horizon, Nc is control horizon, A,B, and C matrices are obtained from the MPC internal model. where $X \in R^{4\times1} A \in R^{4\times4}$, $B \in R^{4\times2}$, $C \in R^{4\times4}$, $\vec{Y} \in R^{4\cdot Np\times1}$, $F \in R^{4\cdot Np\times4}$, $\Phi \in R^{4\cdot Np\times2\cdot Nc}$ and $\vec{U} \in R^{2\cdot Nc\times1}$.

According to the basic theory, MPC calculates input value as a solution to an optimization problem. Therefore, a QP problem is defined as a cost function and constraint

$$\begin{aligned} &\min J = \vec{U}^T \bar{R} \vec{U} + \left(R_s - \vec{Y}\right)^T Q \left(R_s - \vec{Y}\right) \\ &J = \frac{1}{2}\Big(\vec{U}(k)^T 2 \underbrace{(\Phi^T\Phi + \bar{R})}_{H} \vec{U}(k) \\ &+\vec{U}(k)^T \Phi^T \underbrace{4\left(R_s - Fx(k)\right)}_{f} \\ &+2\left(R_s - Fx(k)\right)^T \left(R_s - Fx(k)\right)\Big) \end{aligned} \tag{12}$$

$$M \cdot \vec{U}(k) \leq N \tag{13}$$

where M and N matrices are the combined constraint between input ($\boldsymbol{U}$(k)) and output ($\boldsymbol{Y}$(k)) and are defined by

$$\begin{aligned} &Umin \leq U \leq Umax \\ &\underbrace{\begin{bmatrix} I_{nu} \\ I_{nu} \\ \vdots \\ I_{nu} \end{bmatrix}}_{c2} \underbrace{\begin{bmatrix} U(1)min \\ U(2)min \\ \vdots \\ U(nu)min \end{bmatrix}}_{Umin} \leq \vec{U}(k) \leq \underbrace{\begin{bmatrix} I_{nu} \\ I_{nu} \\ \vdots \\ I_{nu} \end{bmatrix}}_{c2} \underbrace{\begin{bmatrix} U(1)max \\ U(2)max \\ \vdots \\ U(nu)max \end{bmatrix}}_{Umax} \\ &-\vec{U}(k) \leq -c2\ \boldsymbol{Umin} \\ &\vec{U}(k) \leq c2\ \boldsymbol{Umax} \end{aligned} \tag{14}$$

$$\begin{aligned} &Ymin \leq Fx(k) + \Phi\vec{U}(k) \leq Ymax \\ &Ymin - Fx(k) \leq \Phi\vec{U}(k) \\ &\qquad \leq Ymax - Fx(k) \\ &-\Phi\vec{U}(k) \leq -Ymin + Fx(k) \\ &\Phi\vec{U}(k) \leq Ymax - Fx(k) \end{aligned} \tag{15}$$

with

$$\vec{U} = \begin{bmatrix} U(k+1|k) \\ U(k+2|k) \\ \vdots \\ U(k+Nc|k) \end{bmatrix} \tag{16}$$

Car velocity constraint is chosen based on traffic regulations to drive inside the city and the highway. Steering rate constraint is the same as [14]. Output constraint is chosen based on research problems, research limitations, and car physical limitation itself. Y position is bounded to keep the car from going off-road. Heading angle is bounded according to the research limitation that the car cannot do a U-turn. Lastly, steering angle constraint is the same as [14]. This quadratic problem is solved using Hildreth's algorithm.

TABLE I. CONSTRAINT INPUT

Variable Input	Constraint
Velocity	$30\, km/hr \leq V \leq 100\, km/hr$
Steering angle	$-60°/sec \leq \omega \leq 60°/sec$

4) Reference

A time-varying reference of x and y position is used in this research. X position reference is based on the desired velocity and is defined by

$$x(k) = V_{ref} \cdot T_s \cdot k \tag{17}$$

Where Ts is the sampling time and k is the number of iterations. Trajectory tracking tests were done for sinusoidal tracks and a lane-changing maneuver. While obstacle avoidance tests were done by placing objects along a straight-line reference.

III. RESULTS AND DISCUSSION

Five Parameters used in this research are as written in table II. Tests were done by varying the combination of Prediction Horizon (Np), Control Horizon (Nc), and maximum number of iterations for the QP solver. Evaluations were done by comparing the resulting car movement in a 2D plane and comparing the RMSE value of x, y, and absolute position of the car, cost function, and computation time.

TABLE II. SIMULATION PARAMETERS

Parameter	Value
Car Length	4 m
Car Width	2 m
Road Width	4 m
Cost Function Error Weight (Q)	0.4
Cost Function Input Weight (R)	0.6

A. Trajectory Tracking

1) Sinusoidal Track

Sinusoidal track tests were done using 2 variations of velocity, 10 m/s and 20/ m/s. In 10 m/s velocity, the maximum QP iteration used is 40 times and as seen from Fig.7, the car followed the desired sinusoidal reference track. However, there is an x position error of 3.28 m – 4.70 m. in other words, the car failed to reach the desired position at the desired time, there is a 0.32 s – 0.47 s delay to reach the desired position. This delay is caused by the transient time of the car to reach the desired velocity of 10 m/s from 0 m/s. moreover, greater Nc increased the error and cost function due to the future input value used by MPC to solve the QP problem.

The second variation of sinusoidal track test used 20 m/s velocity and 80 times for the maximum number of QP

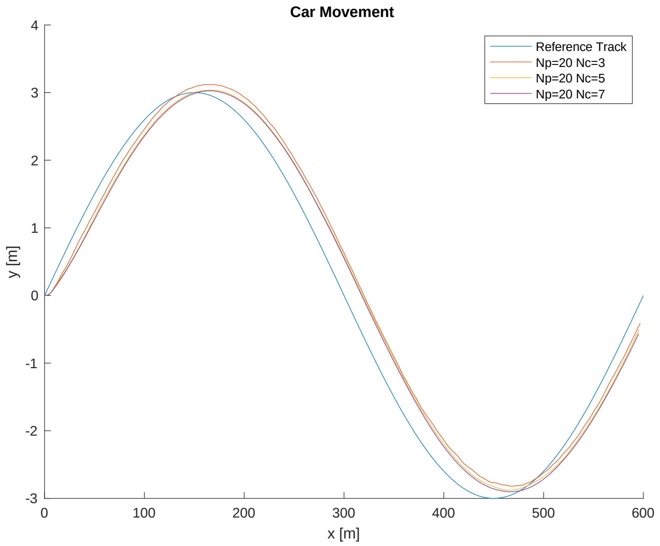

Fig. 5. Sine Wave Tracking with 10 m/s Velocity

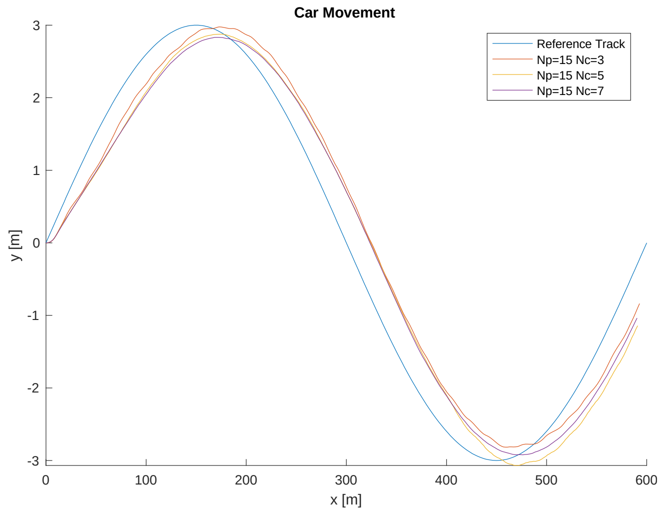

Fig. 6. Sine Wave Tracking with 20 m/s Velocity

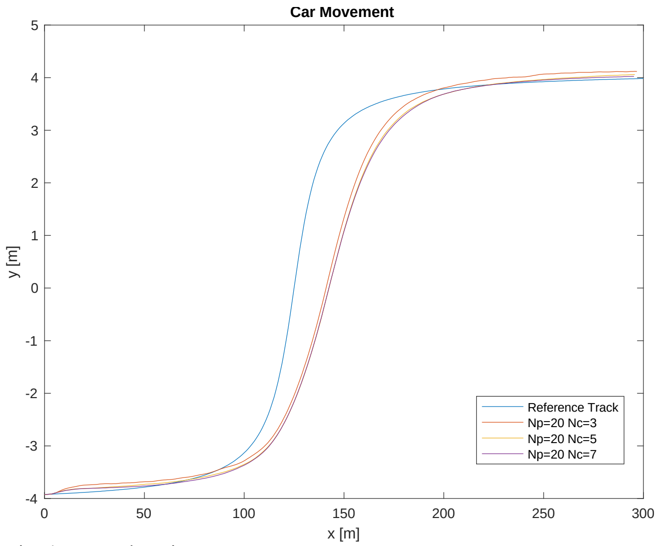

Fig. 4. Lane Changing Maneuver

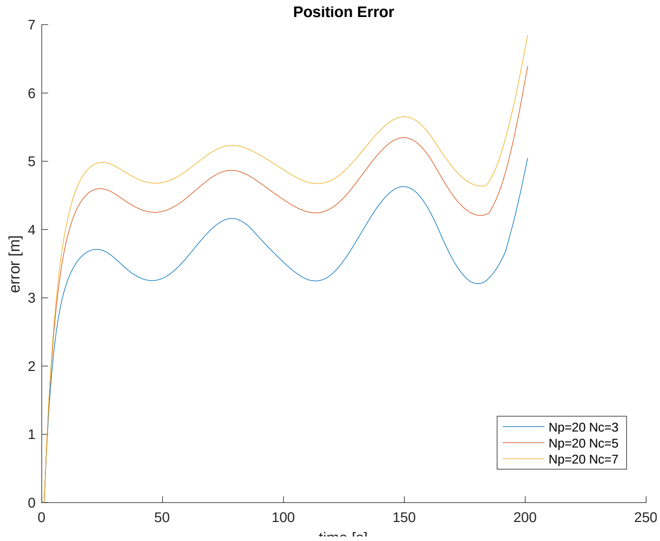

Fig. 7. Error of Sine Wave Tracking with Steering Angle Demand Beyond Constraint

iterations. Similar to the previous test, the car managed to follow the desired track (see Fig.5) with a slight delay of 0.35 s – 0.47 s as seen from the x position error of 7.03 m – 9.46 m. This is caused by the transient time of the car to reach the desired velocity of 20 m/s from 0 m/s.

2) Lane Changing Maneuver

The reference used for this test portrays the movement of a car changing lanes from the middle lane to the left lane. Based on Fig.6, MPC managed to control the car's movement to change lanes. There is an average delay of 0.4 s to reach the desired position at the desired time as seen from the x position error of 3.25 m – 4.66 m. The variation with Nc=7 produced an oscillating movement with a small amplitude before the car moves to change lanes. This means the MPC had a hard time controlling the car to move straight, this suggests that farther Nc doesn’t necessarily produce the best movement.

3) Sinusoidal Track Outside Constraint

The reference used in this test demands a large steering angle beyond the chosen constraint. This test was done to see the limitation of MPC. Results showed the value of error increased over time (see Fig.4). Therefore, it can be implied that MPC was unable to control the car movement to follow the desired track. Other decision that can be made is to decrease the car velocity to maintain the steering angle constraint. However, to produce this decision the problem needs to be described as a path tracking problem, as done in [15] . However, because this research uses trajectory tracking problem, MPC doesn’t have control over car velocity, hence steering control demand is always beyond the predetermined constraint. In trajectory tracking problem, MPC needs to reach the desired position at the desired time, so it controlled the car to go as fast as possible to meet the desired time for the next reference point.

B. Obstacle Avoidance

Obstacle avoidance tests are divided into two parts. Part one is using 1 obstacle and part two is using 2 obstacles. Two velocity variations are used for the test using 1 obstacle.

1) 1 Obstacle

All variations on the first test of obstacle avoidance using 10 m/s velocity result in an oscillating movement (see Fig.9). This can be caused by the combination of parameters chosen was not suitable for a straight-line track. The variation with Np=15 and Nc=3 managed to avoid the obstacle. This was

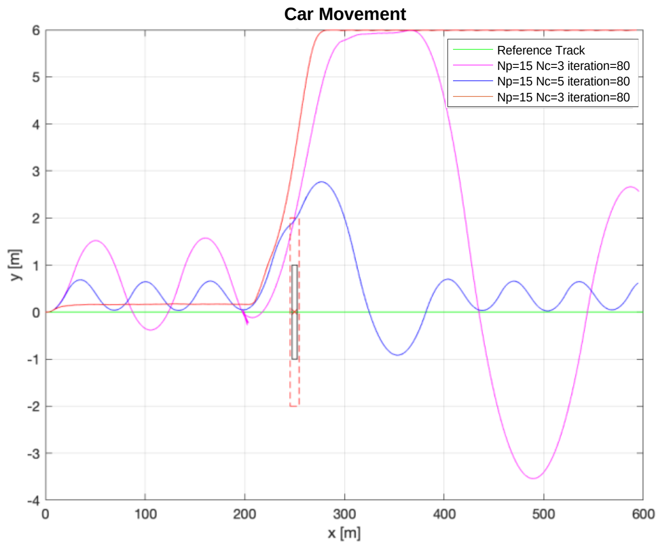

Fig. 9. 1 Obstacle with 10 m/s Velocity

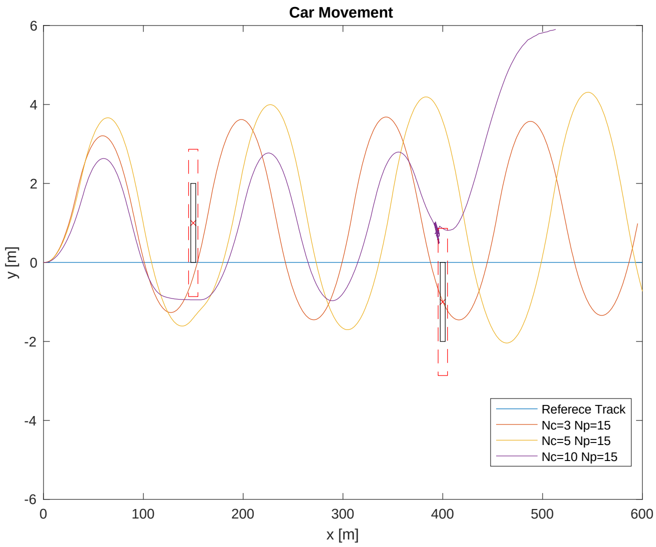

Fig. 8. 2 Obstacles with 10 m/s Velocity

shown by the minimum distance of the car to the obstacle, which was 2.33m, which meets the safe zone criteria. However, avoiding maneuver was ineffective, seen by the unnecessary time taken to go back to the reference track. Moreover, the car movement in this variation oscillated with a big and increasing amplitude after avoiding the obstacle. Therefore, it produced a huge error value. This is maybe due to the number of maximum QP iterations was not enough to find the optimum value of QP problem of MPC.

Meanwhile, the variation with Np=15 and Nc=4 was unable to avoid the obstacle. This was shown by the minimum distance of the car to the obstacle, which was 1.99 m, less than the required 2 m safe zone criteria. The variation with Np=20 and Nc=5 managed to avoid the obstacle but didn't go back to the reference track. There is a possibility that this variation may need more time beyond the simulation time to go back to the track. Apart from that, MPC managed to keep the car inside the lane, fulfilling the Y position constraint of 6 m.

Continuing the test using 1 obstacle with 20 m/s velocity, all variations produced an oscillating movement (see Fig.10). The variation with Nc=7 and a maximum number of QP

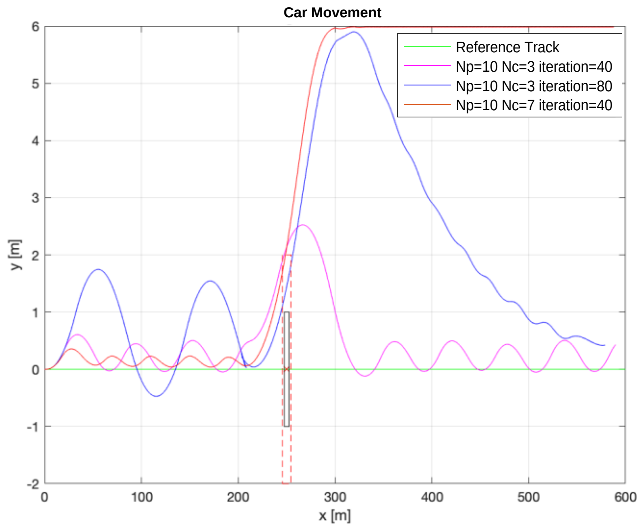

Fig. 10. 1 Obstacle with 10 m/s Velocity

iteration of 40 times showed that MPC was unable to find the optimum input value to minimize lateral error within the constraint. This is due to the further Nc producing a greater cost function, while the QP solver hasn't found the optimum value within 40 iterations. In the case of a combination with Np=10, Nc=7, and a maximum QP iteration of 40 times, it was shown that the car did not move back to the reference trajectory. This also happened in the previous test using 10 m/s velocity, caused by the Np chosen was not far enough or the maximum number of QP iterations being too small to find the optimal value.

2) 2 Obstacles

The next test used 2 obstacles. In this test, the 2 obstacle has a 250 m distance. The car avoiding maneuver was predetermined in the program to turn right on the first obstacle and turn left on the second. Results on Fig.8 showed that all variations produced an oscillating movement with a big amplitude. This means, all the variation has poor straight-line tracking ability. By comparing all variations, the variation with a better obstacle avoidance maneuver has worse straight-line tracking ability.

IV. Conclusion

Based on test results and data analysis, it can be concluded that the LTV MPC with a linear model in this research managed to follow the reference trajectory. The importance of having a well balance combination of Np and Nc value should also be noted to get a good performing MPC design. Lastly, MPC has the limitation where the input demand needs to be within the constraint. Otherwise, QP solver will not be able to find the optimum solution.

As this research was done with several limitations, there is room for improvements. In this regard, the writer suggests considering the following things for future work on this topic:

1. Using a real value of car parameters and car physical constraints for simulation to get a more accurate result.
2. A more complex environment and obstacle model for simulation. This can be done much better by referencing the existing real-world environment.
3. Using sensor for obstacle detection.

4. Using road-relative coordinate (*frenet serret frame* or *curve linear frame*) to improve lateral position accuration.
5. Implementing feedforward configuration to overcome tracking delay.
6. Finding a new combination of MPC parameters to get better performance.
7. Research using a smaller δ value in Hildreth's QP algorithm to ensure that the resulting QP solution is indeed the optimal solution.
8. Taking external disturbance into account in the simulation.

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