Designing an Optimal Control LQT for Controlling and Guidance of Missile

Rusdhianto Effendie AK, Muhamad Rafif Prasetyo, Zulkifli Hidayat
Department of Electrical Engineering, Faculty of Electrical Technology
Sepuluh Nopember Institute of Technology
Surabaya, Indonesia
ditto@ee.its.ac.id, rafif13@mhs.ee.its.ac.id, zulkifli@ee.its.ac.id

***Abstract*—Missile has to be controlled and follow the commanded guidance in order to make its flight hit the target. Since missile has a nonlinear characteristic and coupled dynamic equation, controlling a missile has become more complex. Linear Quadratic Tracking (LQT) is one of optimal control theory where its objective is to make the output of a system tracks its reference as close as possible while minimize or maximize a desired performance index. In this paper, an autopilot for missile is designed which consists nonlinear state feedback decoupler and LQT controller. Pursuit Guidance is used for the guidance law. A missile-target engagement simulation is created using 2 kinds of target; static target and dynamic target. By using static target, the mean of the closest distance between missile and the target is 0.45 meters and by using dynamic target the mean of the closest distance between missile and the target is 2.562 meters.**

Keywords—guided missile; Linear Quadratic Tracking; Nonlinear State Feedback Decoupler; Pursuit Guidance;

I. INTRODUCTION

In military, missile is a rocket which can be controlled directly or automatically to find the target and intercept it. In order to make sure the missile hit the target, missile has to be controlled to always follow reference created by the guidance law. By using aerodynamic forces, missile can make turns when it is flying in the air. The missile can do so by deflecting its fin. Then, by controlling the missile's fin, we can control the missile's movement while it is flying. Difficulty to design the controller arises because missile has nonlinear characteristic and coupled dynamic equation

Optimal control theory is one of modern control theory. The objective in optimal control theory is determining an appropriate control signals that will makes the output of the system satisfy the physical constraints and at the same time, minimize or maximize some given performance index [1]. By looking at the objective, there are 2 kinds of optimal control case; regulator case and tracking case. If the system is a linear system and the desired performance index is in quadratic form, these 2 cases before known as the Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT). Since the missile needs to always follow a certain reference, the desired controller falls into the tracking case.

In order to make the missile, which has nonlinear characteristic and coupled dynamic equation, can be controlled by LQT, which demands a linear system, a nonlinear state feedback decoupler is designed. This decoupler is used to remove the coupled characteristic of the missile and linearize the missile's system.

II. SYSTEM DESIGN

A. *Coordinate System [3]*

Coordinate system adapted in this paper follows the right-handed rule, where the positive x-axis is along the missile's longitudinal axis, the positive y-axis points to the right, and the z-axis is positive to down direction, which is perpendicular with x-axis and y-axis. The earth will be referenced as the inertial coordinate system and the missile will be referenced as the body coordinate system. Earth coordinate system and missile coordinate system respectively will be denoted by (X_e, Y_e, Z_e) and (X_b, Y_b, Z_b). Earth coordinate system and missile coordinate system illustrated in Fig. 1.

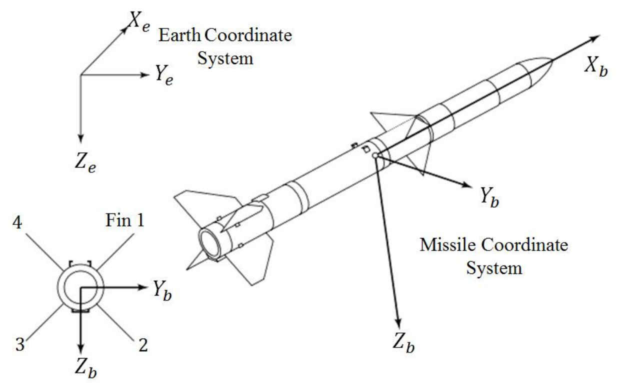

Fig. 1. Earth coordinate system and missile coordinate system

Euler angles is used to represent the missile's attitude in the earth coordinate system. Their notations are φ for roll movement, θ for pitch movement, and ψ for yaw movement.

B. *Missile's Mathematical Model [2],[3]*

The missile's equations of motion derived using Newton's law [2]. Some assumptions are used to model the missile. First, missile has a rigid body, i.e. missile doesn't change in shape and size. Second, aerodynamic effect in the X_b-axis is symmetry. Third, the missile's mass doesn't decrease or increase during its flight, i.e. the missile has a constant mass.

Missile's kinematics equation transforms the missile's translational velocity from body coordinate system to earth coordinate system [3]. This relation is written using Euler angles as seen in (1).

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix}=\begin{bmatrix}c\theta\, c\psi & s\phi\, s\theta\, c\psi - c\phi\, s\psi & c\phi\, s\theta\, c\psi + s\phi\, s\psi\\ c\theta\, s\psi & s\phi\, s\theta\, s\psi + c\phi\, c\psi & c\phi\, s\theta\, s\psi - s\phi\, c\psi\\ -s\theta & s\phi\, c\theta & c\phi\, c\theta\end{bmatrix} \quad (1)$$

Where:

$\dot{x}, \dot{y}, \dot{z}$ = Missile's translational velocity in earth coordinate system
u, v, w = Missile's translational velocity in body coordinate system
c, s, t = Abbreviation of cosine, sine, and tangent

There are 2 kinds of velocity in missile's dynamics equation; translational velocity and rotational velocity. Missile's translational velocity ($\mathbf{V}_b$) is a vector consists of missile's velocity in missile's coordinate system, denoted by u, v, w. In the other hand, missile's rotational velocity ($\boldsymbol{\omega}_b$) is a vector consists of missile's rotational velocity for each axes in body coordinate system, denoted by p, q, r. The missile's velocity magnitude is written in (2).

$$V_m = \|\mathbf{V}_b\| \quad (2)$$

Missile's dynamic equation consists of translational dynamic equation and rotational dynamic equation. The missile's translational dynamic equation states the forces acting on the missile in the body coordinate system so that the missile will experience acceleration. Missile's translational accelerations is written in (3) [2].

$$\left.\begin{aligned}\frac{du}{dt} &= \frac{F_x}{m} + rv - qw\\ \frac{dv}{dt} &= \frac{F_y}{m} + pw - ru\\ \frac{dw}{dt} &= \frac{F_z}{m} + qu - pv\end{aligned}\right\} \quad (3)$$

Where:

F_x, F_y, F_z = Forces acting on the missile in body coordinate system
m = Missile's mass

Forces acting on the missile itself consist of the Thrust, gravitational force, and aerodynamic force [4]. Then, the forces acting on the missile is written in (4).

$$\left.\begin{aligned}F_x &= Thrust - mg\sin\theta + C_x QS\\ F_y &= mg\sin\phi\cos\theta + C_y QS\\ F_z &= mg\cos\phi\cos\theta + C_z QS\end{aligned}\right\} \quad (4)$$

Where:

$Thrust$ = Forces acting on the missile in body coordinate system
g = Gravitational acceleration
Q = Dynamic pressure ($\frac{1}{2}\rho V_m^2$)
ρ = Air density
S = Missile's surface area
C_x = Aerodynamic drag force coefficient
C_y = Aerodynamic side force coefficient
C_z = Aerodynamic lift force coefficient

The aerodynamic force coefficients (C_x, C_y, C_z) are affected by Angle of Attack, Sideslip angle, Mach number, and the missile's fin deflection angle [4]. These coefficients then can be written in (5).

$$\left.\begin{aligned}C_x &= C_{x1}\alpha + C_{x2}M\\ C_y &= C_{y1}\beta + C_{y2}M + C_{y3}\delta_r\\ C_z &= C_{z1}\alpha + C_{z2}M + C_{z3}\delta_e\end{aligned}\right\} \quad (5)$$

Where:

α = Angle of Attack ($\tan^{-1}\frac{w}{u}$)
β = Sideslip angle ($\sin^{-1}\frac{v}{V_m}$)
M = Mach number ($\frac{V_m}{340}$)
δ_r = rudder deflection angle
δ_e = Elevator deflection angle

Missile's rotational acceleration on each axes in body coordinate system is written in (6) [2]. From (6), the aileron (δ_a), rudder (δ_r), and elevator (δ_e) is the missile system's input.

$$\left.\begin{aligned}\frac{dp}{dt} &= \frac{QSd(C_{l1}\alpha + C_{l2}\beta + C_{l3}p + C_{l4}\delta_a) + (I_y - I_x)qr}{I_x}\\ \frac{dq}{dt} &= \frac{QSd(C_{m1}\alpha + C_{m2}q + C_{m3}\delta_e) + (I_z - I_x)pr}{I_y}\\ \frac{dr}{dt} &= \frac{QSd(C_{n1}\beta + C_{n2}r + C_{n3}\delta_r) + (I_x - I_y)pq}{I_z}\end{aligned}\right\} \quad (6)$$

Where:

d = Missile's diameter
I_x, I_y, I_z = Missile's moment of inertia on each axes in body coordinate system

C. Missile's Autopilot

The purpose of the missile's control system or also known as missile's autopilot is to ensure the missile's stability and performance. The autopilot also has to fly the missile right to the designed location or straight to the target according to the signal generated by the guidance law used.

1) Nonlinear State Feedback Decoupler [5]

In state-space form, a nonlinear and coupled system can be written as in (7).

$$\left.\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{G}(\mathbf{x},t)+\mathbf{F}(\mathbf{x},t)\mathbf{u}\\ \mathbf{y} &= \mathbf{C}\mathbf{x}\end{aligned}\right\} \tag{7}$$

Where:

$\mathbf{x}$ = state vector
$\mathbf{u}$ = Input vector
$\mathbf{G}(\mathbf{x},t)$ = Nonlinear state matrix
$\mathbf{F}(\mathbf{x},t)$ = Nonlinear input matrix
$\mathbf{y}$ = Output vector
$\mathbf{C}$ = Output matrix

Define a new state matrix $\mathbf{A}$ which has constant form and decoupled, and a new input matrix $\mathbf{B}$ which also has a constant form. A new state-space can be constructed from using these new matrixes in the form (8).

$$\dot{\mathbf{x}}(t) = \mathbf{G}(\mathbf{x},t)+\mathbf{F}(\mathbf{x},t)\mathbf{u}+\mathbf{A}\mathbf{x}-\mathbf{A}\mathbf{x}+\mathbf{B}\hat{\mathbf{u}}-\mathbf{B}\hat{\mathbf{u}} \tag{8}$$

Where $\hat{\mathbf{u}}$ is the new input vector. In order to make the system has a linear and decoupled form, then (9) has to be satisfied.

$$\mathbf{G}(\mathbf{x},t)+\mathbf{F}(\mathbf{x},t)\mathbf{u}-\mathbf{A}\mathbf{x}-\mathbf{B}\hat{\mathbf{u}}=\mathbf{0} \tag{9}$$

If $\mathbf{F}(\mathbf{x},t)$ has an inverse or pseudoinverse, it is possible to make a linear and decoupled system by using (10), a new input into the system. The state-space diagram of the system using nonlinear state space feedback decoupler can be seen in Fig. 2.

$$\mathbf{u}=\mathbf{F}(\mathbf{x},t)^{-1}(\mathbf{A}\mathbf{x}+\mathbf{B}\hat{\mathbf{u}}-\mathbf{G}(\mathbf{x},t)) \tag{10}$$

Since the desired controlled variable is missile's heading, the equation which will become the system is the missile's rotational dynamic equation. Decoupling process is done by feeding back the parameters which cause the missile has nonlinear characteristic. Linearization process is done by define new parameters so that the rotational dynamic equation resembles first order response. The new time constant must be reasonable enough for the system or else the system would break down. In order to reduce the likeability that the missile miss its target, the missile's fins must have a fast response. Here the desired time constant is 0.1 seconds, for each axes in body coordinate system. Then, the new input for the rotational dynamic equation is (11).

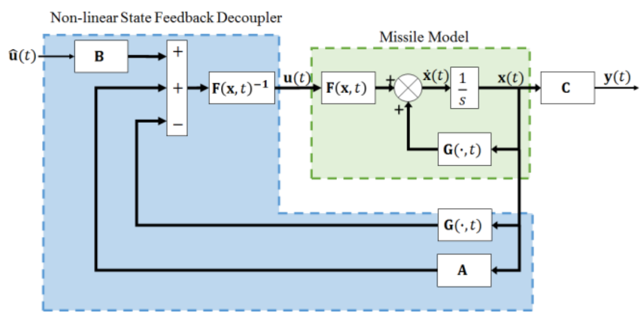

Fig. 2. System with nonlinear state feedback decoupler

$$\left.\begin{aligned}
\delta_a &= -\frac{C_{l1}}{C_{l4}}\alpha-\frac{C_{l2}}{C_{l4}}\beta-\frac{C_{l3}}{C_{l4}}p-\frac{(I_y-I_z)qr}{C_{l4}QSd}\\
&\quad -\frac{\lambda_p I_x p}{C_{l4}QSd}+\frac{\lambda_p I_x U_p}{C_{l4}QSd}\\
\delta_e &= -\frac{C_{m1}}{C_{m3}}\alpha-\frac{C_{m2}}{C_{m3}}q-\frac{(I_z-I_x)pr}{C_{m3}QSd}\\
&\quad -\frac{\lambda_q I_y q}{C_{m3}QSd}+\frac{\lambda_1 I_y U_q}{C_{m3}QSd}\\
\delta_r &= -\frac{C_{n1}}{C_{n3}}\beta-\frac{C_{n2}}{C_{n3}}r-\frac{(I_x-I_y)pq}{C_{n3}QSd}\\
&\quad -\frac{\lambda_r I_z r}{C_{n3}QSd}+\frac{\lambda_r I_z U_r}{C_{n3}QSd}
\end{aligned}\right\} \tag{11}$$

Where:

$\lambda_p=\frac{1}{\tau_p}=10,\quad \tau_p=$ new time constant for roll rate

$\lambda_q=\frac{1}{\tau_q}=10,\quad \tau_q=$ new time constant for pitch rate

$\lambda_r=\frac{1}{\tau_r}=10,\quad \tau_r=$ new time constant for yaw rate

$U_p, U_q, U_r=$ The new input for the linearized system

2) Linear Quadratic Tracking (LQT) [6]

The objective of LQT is to control the system so that the output of the system follows the desired output as close as possible while minimize or maximize some desired index performance. The steps to design a system using LQT is given below.

a. Get the system's state matrix **A**, system's input matrix **B**, and system's output matrix **C**.

b. Determine the order and the values of matrix **Q** and matrix **R**.

c. Find **P**, the solution of Riccati Algebraic Equation, using (12). The apostrophe sign means a transposed matrix.

$$\mathbf{A}'\mathbf{P}+\mathbf{P}\mathbf{A}+\mathbf{C}'\mathbf{Q}\mathbf{C}-\mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}'\mathbf{P}=\mathbf{0} \tag{12}$$

d. Create Kalman Gain matrix using (13).

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}'\mathbf{P} \qquad (13)$$

e. Create Following Model using (14).

$$\dot{\mathbf{V}}(t) = -(\mathbf{A} - \mathbf{BK})'\mathbf{V}(t) - \mathbf{C}'\mathbf{Qr}(t) \qquad (14)$$

f. The optimal control signal which become the system's input is given by (15).

$$\mathbf{u}^*(t) = -\mathbf{Kx}(t) + \mathbf{R}^{-1}\mathbf{B}'\mathbf{V}(t) \qquad (15)$$

A state-space diagram of a system with LQT can be seen in Fig. 3.

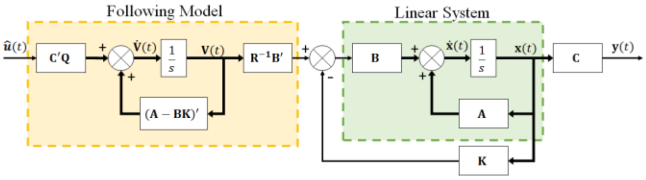

Fig. 3. A system with LQT

Since the input of the system has been modified and become (11), the missile system has become linearized and decoupled. Now LQT can be used to become the controller for the missile model. The linearized missile model is designed in state-space form. The desired controlled variable included in the linearized missile model. The resulted state-space model can be seen in (16).

$$\left.\begin{aligned}
\begin{bmatrix}\dot{\phi}\\ \dot{p}\\ \dot{\theta}\\ \dot{q}\\ \dot{\psi}\\ \dot{r}\end{bmatrix} &= \begin{bmatrix}0&1&0&0&0&0\\0&\lambda_p&0&0&0&0\\0&0&0&1&0&0\\0&0&0&\lambda_q&0&0\\0&0&0&0&0&1\\0&0&0&0&0&\lambda_r\end{bmatrix}\begin{bmatrix}\phi\\p\\\theta\\q\\\psi\\r\end{bmatrix}\cdots\\
&+\begin{bmatrix}0&0&0\\\lambda_p&0&0\\0&0&0\\0&\lambda_q&0\\0&0&0\\0&0&\lambda_r\end{bmatrix}\begin{bmatrix}U_p\\U_q\\U_r\end{bmatrix}\\
\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} &= \begin{bmatrix}0&1&0&0&0&0\\0&0&0&1&0&0\\0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}\phi\\p\\\theta\\q\\\psi\\r\end{bmatrix}
\end{aligned}\right\} \qquad (16)$$

The missile needs to always follow the reference given in order to reduce the likeability to miss the target. So the designed matrix **Q** and matrix **R** is (17). The determinant of **Q** has to be big to reduce the error between the actual output (missile's actual heading) and the desired output (missile's desired heading). Matrix **R** left to be identity since we don't focus to reduce the energy consumption.

$$\left.\begin{aligned}\mathbf{Q} &= \begin{vmatrix}1000&0&0\\0&1000&0\\0&0&1000\end{vmatrix}\\ \mathbf{R} &= \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\end{aligned}\right\} \qquad (17)$$

D. *Pursuit Guidance*

Pursuit Guidance will make the missile's heading always pointing to the target after it is being launched, so the missile flies directly toward its target at all times. This guidance law will always try to remove the relative Line of Sight (LOS) angle between the missile and its target. Suppose the target's position in earth coordinate system is denoted by x_t, y_t, z_t and missile's position in earth coordinate system is denoted by x_m, y_m, z_m. Then the relative position of target to the missile is given in (18). The relative angle created between missile and the target can be seen at Fig. 4.

$$\left.\begin{aligned}x_r &= x_t - x_m\\ y_r &= y_t - y_m\\ z_r &= z_t - z_m\end{aligned}\right\} \qquad (18)$$

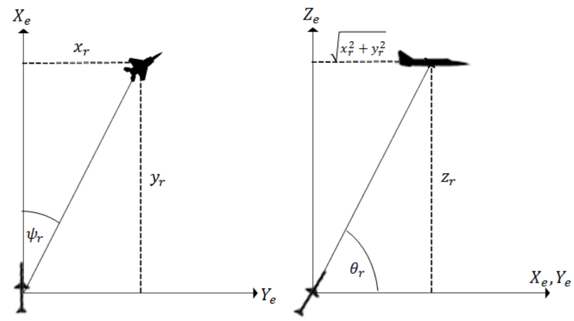

Fig. 4. Relative angle between missile and the target

If the missile's position and the target's position are known very good, then the relative angle between the missile and the target can be found using simple trigonometry rule shown in (19) and (20).

$$\theta_r = \tan^{-1}\left(\frac{z_r}{\sqrt{x_r^2 + y_r^2}}\right) \tag{19}$$

$$\psi_r = \tan^{-1}\left(\frac{y_r}{x_r}\right) \tag{20}$$

Where:

θ_r = Relative pitch angle between missile and the target in X_e vs Y_e plane.

ψ_r = Relative yaw angle between missile and the target in X_e,Y_e vs Z_e plane

E. Target's Position Prediction

One disadvantage using Pursuit Guidance is the maneuver required by the missile increases as the missile comes closer to the target, and at the end of flight, missile has to make the sharpest maneuver so that it can intercept the target. Since the missile has limited maneuver capability, it is better to make the missile fly to the expected intercept position. After the missile launched, missile will try to predict where the target's position at some time in the future by using the past target's positions. The output of this prediction will become input for the guidance.

The missile gets information about target's position every t_s seconds. The target's position at $x(k-2)$, $x(k-1)$, and $x(k)$, where $x(k-2)$ is the first time target's position known by the missile, are located using (21), or in matrix form in (22).

$$\left.\begin{aligned} x(k-2) &= p_t + v_t t_s + a_t t_s^2 \\ x(k-1) &= p_t + 2v_t t_s + 4a_t t_s^2 \\ x(k) &= p_t + 3v_t t_s + 9a_t t_s^2 \end{aligned}\right\} \tag{21}$$

$$\begin{bmatrix} x(k-2) \\ x(k-1) \\ x(k) \end{bmatrix} = \begin{bmatrix} 1 & t_s & t_s^2 \\ 1 & 2t_s & 4t_s^2 \\ 1 & 3t_s & 9t_s^2 \end{bmatrix} \begin{bmatrix} p_t \\ v_t \\ a_t \end{bmatrix} \tag{22}$$

Where:

p_t = Target's position

v_t = Target's velocity

a_t = Target's acceleration

By using the information about target's positions and (21), the target's position, velocity, and acceleration at $x(k-2)$ can be determined using (22).

$$\begin{bmatrix} p_t \\ v_t \\ a_t \end{bmatrix} = \begin{bmatrix} 1 & t_s & t_s^2 \\ 1 & 2t_s & 4t_s^2 \\ 1 & 3t_s & 9t_s^2 \end{bmatrix}^{-1} \begin{bmatrix} x(k-2) \\ x(k-1) \\ x(k) \end{bmatrix} \tag{22}$$

Then the target's position at $k+1$, or t_s second in the future, can be predicted using (23).

$$x(k+1) = p_t + 4v_t t_s + 16a_t t_s^2 \tag{23}$$

Since the target's velocity and the missile's velocity are known, new parameter called closing velocity can be determined using (24). Then a parameter called time-to-go, i.e. the time required for the missile to intercept its target, can be determined using (25). Finally, the intercept location between missile and the target can be predicted by (26).

$$V_c = \left\| \begin{matrix} v_{tx} - v_{mx} \\ v_{ty} - v_{my} \\ v_{tz} - v_{mz} \end{matrix} \right\| \tag{24}$$

$$t_{tgo} = \frac{\sqrt{x_r^2 + y_r^2 + z_r^2}}{V_c} \tag{25}$$

$$x(t_{tgo}) = p_t + v_t(t_{tgo}) + a_t(t_{tgo})^2 \tag{26}$$

Where:

V_c = Closing velocity

v_{tx}, v_{ty}, v_{tz} = Target's velocity respectively on X_e-axis, Y_e-axis, and Z_e-axis.

v_{mx}, v_{my}, v_{mz} = Missile's velocity respectively on X_e-axis, Y_e-axis, and Z_e-axis.

t_{tgo} = Time-to-go.

III. Simulation Results

The simulation run using 2 kinds of target; nonmaneuvering target and maneuvering target. Simulation using nomaneuvering target consists of 3 cases, each have 3 scenarios, while simulation using maneuvering target consists of 3 cases, each with 2 scenarios. Some initial parameters set for the simulations are the missile has Thrust force 8000 Newton for 5 seconds and decreased to 4000 Newton for the next 45 seconds, missile launched from coordinate (0,0,0) in earth coordinate system, missile fin's deflection is limited from -60 degrees to +60 degrees, missile's initial velocity and acceleration respectively are 0 m/s and 0 m/s^2, the value of air density and gravity acceleration are constant, regardless how high the missile is, and the simulation is stopped when the missile reached the closest distance to the target.

A. Nonmaneuvering Target

The objective of simulation using nonmaneuvering target is to see how target's distance affects the closest distance between missile-target. Initial heading (ϕ_0, θ_0, ψ_0) in this simulation is (0 45 45) degrees. Initial positions of the target are shown in Table 1. The resulted simulation graph is shown in Fig. 5, Fig. 6, and Fig. 7. The closest distance between missile-target is shown in Table 2.

TABLE I. INITIAL POSITION FOR NONMANEUVER TARGET SIMULATION

Scenario	Initial Position [X_e Y_e Z_e]	Note
1	[1000 500 500]	Target changes position on X_e-axis, the others don't change.
2	[2000 500 500]	
3	[3000 500 500]	
4	[500 1000 500]	Target changes position on Y_e-axis, the others don't change.
5	[500 2000 500]	
6	[500 3000 500]	
7	[500 500 1000]	Target changes position on Z_e-axis, the others don't change.
8	[500 500 2000]	
9	[500 500 3000]	

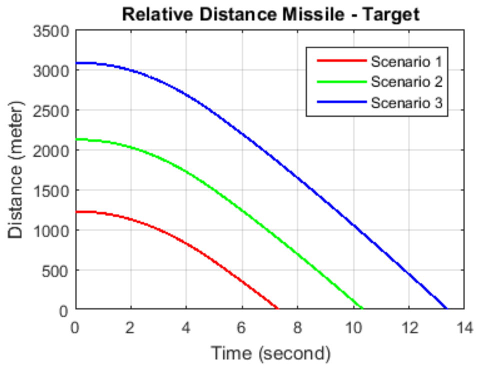

Fig. 5. Simulation with nonmaneuvering target case 1

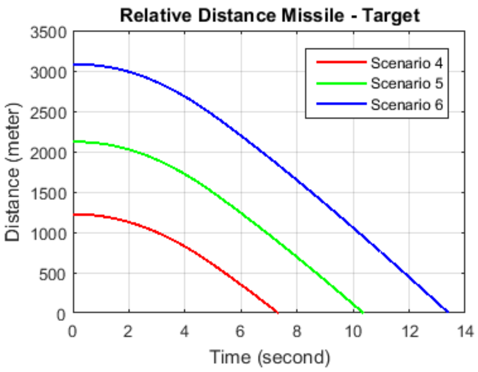

Fig. 6. Simulation with nonmaneuvering target case 2

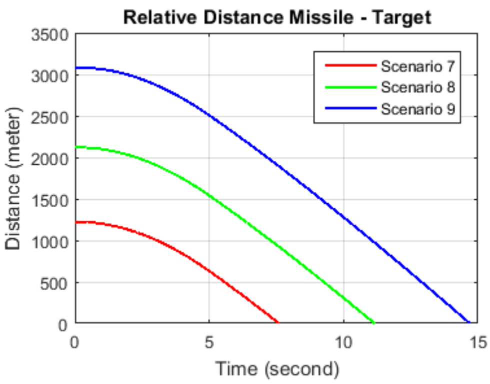

Fig. 7. Simulation with nonmaneuvering target case 3

TABLE II. SIMULATION RESULT WITH NONMANEUVERING TARGET

Scenario	Flight Duration (seconds)	Closest Distance (meter)
1	7.32	0.3718
2	10.35	0.6101
3	13.40	0.2594
4	7.33	0.6593
5	10.37	1.1940
6	13.41	0.0440
7	7.60	0.4383
8	11.50	0.0863
9	14.70	0.3627
Average	**10.66**	**0.4473**

By looking at the Fig. 5, Fig.6, and Fig.7, in all scenarios the missile moves towards the target. It is seen by the graph that the relative distance between missile-target is decreasing while the time goes on. By looking at Table 2, the closest distance created between missile and the target is close. It can be seen from the average closest distance between missile-target is less than 0.5 meters. Then it can be concluded that the designed autopilot for the missile works good with nonmaneuvering target.

B. *Maneuvering Target*

The objective of simulation using maneuvering target is the same as the nonmaneuvering target before. Initial heading (ϕ_0, θ_0, ψ_0) for this simulation is (0 60 0) degrees. Initial positions of the target are shown in Table 3. The resulted simulation graph is shown in Fig. 8, Fig. 9, and Fig. 10. The closest distance between missile-target is shown in Table 4.

TABLE III. INITIAL CONDITIONS FOR MANEUVERING TARGET SIMULATION

Scenario	Initial		
	Position (m)	*Velocity (m/s)*	*Acceleration (m/s^2)*
1	[1000 1000 1000]	[0 -50 0]	[0 0 0]
2	[1000 1000 1000]	[0 -100 0]	[0 0 0]
3	[1500 0000 0000]	[-50 0 40]	[0 0 0]
4	[1500 0000 0000]	[-50 0 40]	[-5 0 0]
5	[1000 1000 1000]	[-50 -50 30]	[0 0 0]
6	[1000 1000 1000]	[-50 -50 30]	[8 8 0]

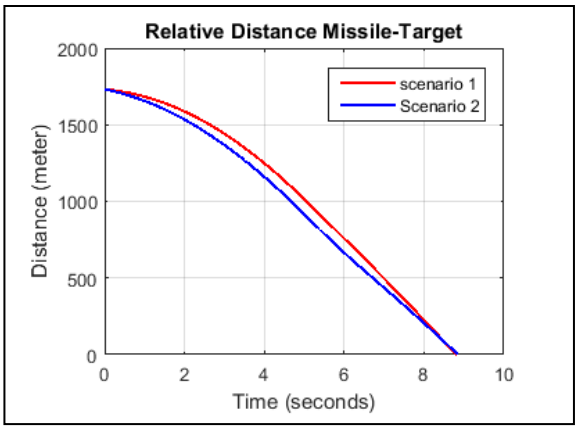

Fig. 8. Simulation with maneuvering target case 1

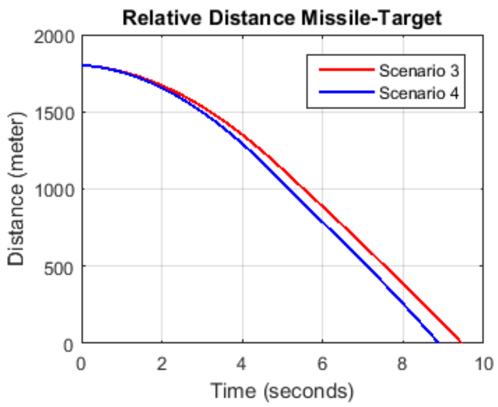

Fig. 9. Simulation with maneuvering target case 2

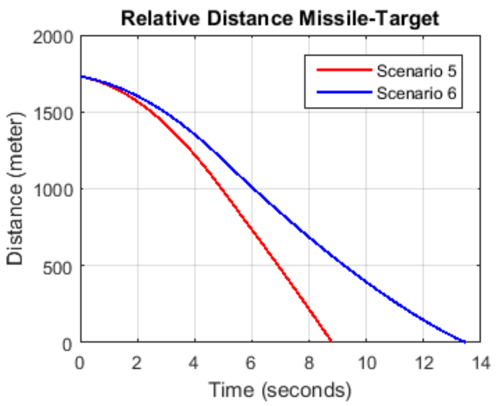

Fig. 10. Simulation with maneuvering target case 3

TABLE IV. SIMULATION RESULT WITH MANEUVERING TARGET

Scenario	Flight Duration (seconds)	Closest Distance (meter)
1	8.81	1.4060
2	8.85	0.6871
3	9.46	2.1270
4	8.89	4.3470
5	8.81	2.7220
6	13.47	4.0820
Average	**9.715**	**2.5619**

By looking at Fig.9 and Fig. 10, the missile fly towards the target. Looking at Fig. 11, the missile seems have difficulty to reach the target, but eventually it did. This could happen because the target has relatively high initial acceleration. Then it is possible the missile could not hit the target if the target has faster velocity than the missile itself. By looking at Table 4, the average closest distance created between missile and the target is 2.562 meters. It is worse compared with the simulation with nonmaneuvering target. Flight duration in scenario 6 is relatively longer than other scenarios. Looking at the initial conditions, this could happen because the target is moving away from the missile while having acceleration. However, the target still can be intercepted.

IV. CONCLUSION

The missile system which basically has nonlinear and coupled characteristics can be linearized and decoupled using nonlinear state feedback decoupler. After being linearized, LQT is connected to the new system to become the controller. It is shown from the simulation results that the designed autopilot works well, proven by the average closest distance between missile-target for nonmaneuvering target is 0.45 meters, and for maneuvering target is 2.56 meters.

REFERENCES

[1] B. Özkan, “Dynamic Modeling, Guidance, and Control of Homing Missiles,” Dec. 2017.

[2] G. M. Siouris, *Missile Guidance and Control Systems*. Springer New York, 2010.

[3] G. M. Siouris, *Aerospace Avionics Systems: A Modern Synthesis*. Academic Press, 1993.

[4] P. H. Zipfel, *Modeling and Simulation of Aerospace Vehicle Dynamics*. American Institute of Aeronautics and Astronautics, 2007.

[5] A. K. R. Effendi, M. Rameli, E. Iskandar, and M. Baihaqi, “Linearization and decoupling controller for quadruple tank,” in *2017 International Seminar on Intelligent Technology and Its Applications (ISITIA)*, 2017, pp. 233–237.

[6] F. L. Lewis, D. Vrabie, and V. L. Syrmos, *Optimal Control*, Third. New Jersey: John Wiley & Sons, 2012.

[7] W. Bużantowicz, “Matlab Script for 3D Visualization of Missile and Air Target Trajectories,” vol. 5, pp. 419–422, Sep. 2016.