PAPR Reduction of CC-OFDM MIMO Radar Waveform using Golay Codes

Devy Kuswidiastuti*
Department of Electrical Engineering
Institut Teknologi Sepuluh Nopember
Surabaya, Indonesia
devy@its.ac.id

Titiek Suryani
Department of Electrical Engineering
Institut Teknologi Sepuluh Nopember
Surabaya, Indonesia
titiek@its.ac.id

Alif Yudha Prasetya
Department of Electrical Engineering
Institut Teknologi Sepuluh Nopember
Surabaya, Indonesia
alifyudhaprasetya@gmail.com

Abstract— **The development of modern radar technology demands multi-functional features that lead to the importance of designing a flexible, software-controlled digital waveform generator. The CC-OFDM MIMO radar waveforms allow flexibility in subcarrier management; hence it is possible to generate multiple narrow beams which transmitted simultaneously for providing all time coverage of the area.**

However, the main issue of OFDM is the high level of its PAPR value. This will cause non-linear distortion that potentially ruin the orthogonality of the OFDM subcarriers and in the end gives error in the radar detection results. In this paper, PAPR reduction using phase coding is done, by comparing the performance of complementary Hadamard, complementary Golay codes and combining it with Selective Mapping (SLM). The PAPR results show that complementary Golay codes without SLM are superior to complementary Hadamard with or without SLM. The maximum PAPR value of 65 beams transmitted simultaneously are 4.9 dB and 11.1 dB for CC OFDM waveform using complementary Golay codes and Hadamard respectively.

Keywords—codes, golay, hadamard, OFDM, PAPR, SLM.

I. INTRODUCTION

The research on the modern radar waveform has been rapidly increased over the years since the idea of combining radar and communication systems. The multifunctional radar has been one of the requirements of the modern radar. Hence, OFDM has been discussed as a potential candidate because it is used in communication nowadays and it has some unique characteristics that are very useful for radar system. The orthogonality between the OFDM subcarrier brings a huge advantage to radar detection performance. MIMO radar using OFDM waveform gives the possibility of multibeam transmission with circular code on its transmitters and by using sub bands division and allocation, as well as codes allocation schedule for producing the multibeam signals [1]. Because of the orthogonality between the OFDM subcarriers, the radar detection will provide a range and Doppler profile which are independent of each other, unlike the case when LFM signals are used.

However, the advantages come with a challenging problem, because of the nature characterized by its high peak to average power ratio (PAPR). This will potentially cause problems because the high peaks can cause a non-linear distortion when it passes through the power amplifiers[2-3]. High PAPR signal required a power amplifier with a wide dynamic range to handle the peak power levels. It will add more complexity and cost to design such amplifiers. High PAPR signals can cause interference to the neighboring channels due to the nonlinear effects. Hence, it is better to avoid the nonlinear effects, by reducing the PAPR of the signals transmitted through the power amplifiers.

There are several techniques which are commonly used to reduce the PAPR of the OFDM signals, such as clipping and filtering [4], PTS [5-9], coding [10-14], selected mapping (SLM) [12–13], etc. From those options, clipping is not the best solution because clipping itself is a nonlinear process and it may cause in-band and out-of-band interferences that will destroy the orthogonality between the OFDM subcarriers. Another option is by using phase coding of the OFDM subcarriers that must result in the lowest possible PAPR.

In consideration of orthogonalization of OFDM symbols as previously discussed, the codes of choice must meet the two objectives of orthogonalizing the symbols and producing a low PAPR. Two types of codes considered for these purposes include Hadamard and Golay codes, which are subsequently compared, including the cases in which the Selective Mapping (SLM) technique is implemented. In this paper, the evaluation of these codes implemented in OFDM MIMO radar waveforms are discussed, from which the Golay codes, even without the SLM, emerge as the best one that fulfills the two objectives. Hence, Golay codes can be considered the best option in these cases. In fact, Golay codes have been used in [14] to lower the PAPR of the OFDM signal.

II. CC-OFDM MIMO RADAR SIGNAL AND SYSTEM

The array factor (AF) of the CC-OFDM waveform as a function of angle θ, subcarrier frequency f_n, and time delay Δt_b for phase steering of the b-th beam is given by:

$$AF(\theta, f_{n,a,b}, \Delta t_b) = \sum_{p=-\frac{(N_t-1)}{2}}^{\frac{(N_t-1)}{2}} \underbrace{\underbrace{C_{n,p,a,b}\, I_p\, e^{jp2\pi f_{n,a,b}\Delta t_b}}_{\text{the waveform effect}} \; \underbrace{e^{jp\alpha(\theta)}}_{\text{the array radiation effect}}}_{\text{steering vector, } sv_p(\theta, f_{n,a,b}, \Delta t_b)} \quad (1)$$

With $\beta(f_{n,a,b}\,\Delta t_b) = 2\pi f_{n,a,b}\Delta t_b$ denotes the phase difference between the array antennas. $n_{a,b}$ denotes the n-th subcarrier used by the a-th symbol of the b-th beam, and $f_{n,a,b} = n_{a,b}\Delta f$ is the subcarrier frequency in Hertz. $C_{n,p,a,b}$ is the phase correction given to the n-th subcarrier used for the a-th symbol of the b-th beam which is

How to cite: Kuswidiastuti, D., Suryani,T., Prasetya, A.Y. (2024). PAPR Reduction of CC-OFDM MIMO Radar Waveform using Golay Codes. *JAREE (Journal on Advanced Research in Electrical Engineering), 8(2).*

transmitted from the p-th transmitter. The array factor formula in Eq. (1) shows two terms related to the waveform effect and the array radiation effect. The array radiation effect will depend on the physical array configuration, while the other one depends on the waveform design. The OFDM waveform generated in transmitter p, for the a-th symbol of the b-th beam is defined as follows:

$$x_{p,a,b}^{\mathbf{ofdm}}(t) = \sum_{n=0}^{N_c-1} g_{n,a,b} e^{j2\pi f_{n,a,b}(t-(a-1)T_0)} \quad (2)$$

$x_{p,a,b}^{\mathbf{ofdm}}(t)$ is a-th symbol of the Golay coded OFDM signal of the b-th beam which is transmitted through the p-th transmitter. When N_t elements are used for transmitting the signals then the CC-OFDM transmit signal in baseband will be:

$$x_{p,a,b}^{\mathbf{cc-ofdm}}(t) = sv_p\left(\theta, f_{n,a,b}, \Delta t_b\right) x_{a,b}^{\mathbf{ofdm}}(t) \quad (3)$$

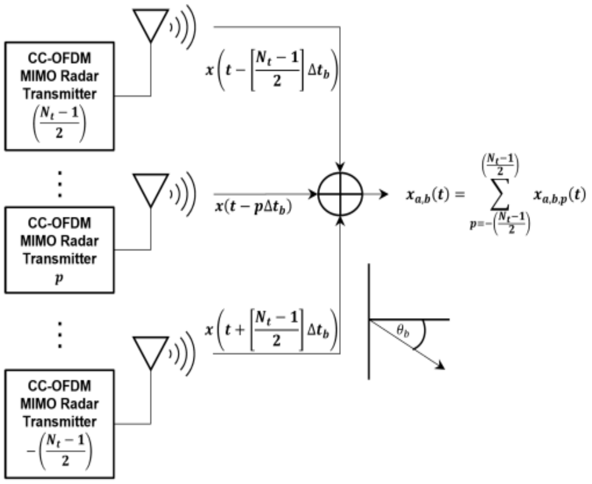


Fig. 1 CC-OFDM MIMO radar signal transmission.

Fig. 1 shows the CC-OFDM MIMO Radar transmitters, with N_t number of transmitters, and the zero-phase reference is in the center of the array. Each transmitter element p will transmit CC-OFDM waveform, $x(t-p\Delta t)$, which basically the same as waveform as $x(t)$, only it has a small-time delay difference between its successive elements. This time delay difference represents the phase difference required by the antenna array elements to create beam towards a specific direction.

In this paper, the radar waveform specifications are specified in Table 1. The radar waveform is specified for a long-range surveillance radar, with 65 beams for coverage from -45 to 45 degrees. The total number of subcarriers are divided into 8 sub bands with 256 subcarriers per sub band. The 65 beams are transmitted using these 8 sub bands, which means each sub band is responsible for transmitting 8 beams on average.

The sub band and codes allocation for the beams are crucial for maintaining the orthogonality between the OFDM subcarriers and also keeping the PAPR low in such a way that it will not cause non-linearity distortion on the power amplifier.

Table 1 Radar Waveform Specifications for long range surveillance

Specifications		Unit
Carrier frequency	3	GHz
Bandwidth	20	MHz
Number of subcarriers	2048	Subcarriers
Number of sub bands	8	Sub bands
Number of sub carriers per sub band	256	Subcarriers
Number of beams	65	beams
OFDM symbols duration	0.1	ms
Subcarrier spacing	10	kHz

The OFDM signal generator is basically an IFFT operation, which provides OFDM with flexibility on its subcarrier's allocation. It will be a benefit for CC OFDM, because the sub band allocation of the beams can be arranged so that it could be used for transmitting multi beams simultaneously.

Fig. 2 shows the digital waveform generator for single beam, for example, beam -31 will be transmitted using sub band 1, then a code with the same length with the number of subcarriers per sub band is needed for coding the subcarriers phase of sub band 1. Based on the waveform specification in Table 1, the number of subcarriers per sub band is 256 subcarriers. Hence the code length must be 256 bits consisting of -1, or 1. According to [1], the center main lobe of beam -31 is directed to the angle of $\theta_b = -43.1°$. For generating only beam -31, a specified code is used to code the subcarriers in sub band 1 which are used for transmitting beam 31, and the other subcarriers are set to 0. The coded OFDM subcarriers are the undergo an IFFT procedure to obtain the time domain representation of the OFDM signal. After IFFT, the time domain OFDM signal is then digitally converted to an IF frequency of 20MHz.

By using a specified scheduling as introduced in [1], for the sub band and the code for the beam, the same procedure can be used for generating each individual beam. It means there will be 65 beam signals which are stored and served as the radar signal references. Multiple and simultaneous beams can be obtained by adding up the 65 individual signals and transmitting them using the 8 available sub bands. Because the 65 beam signals are coded using orthogonal codes, even though they are transmitted simultaneously, the reflected signal coming from the targets can be distinguished by the radar receiver.

In this paper, two orthogonal codes, i.e., Golay and Hadamard codes are analyzed, in order to find the most suitable codes which can fulfill the requirement of CC-OFDM MIMO radar. The two important characteristics that the code must have is orthogonality and that it can lower the PAPR of the OFDM signal. The code generators for Golay and Hadamard codes are discussed in the next section.

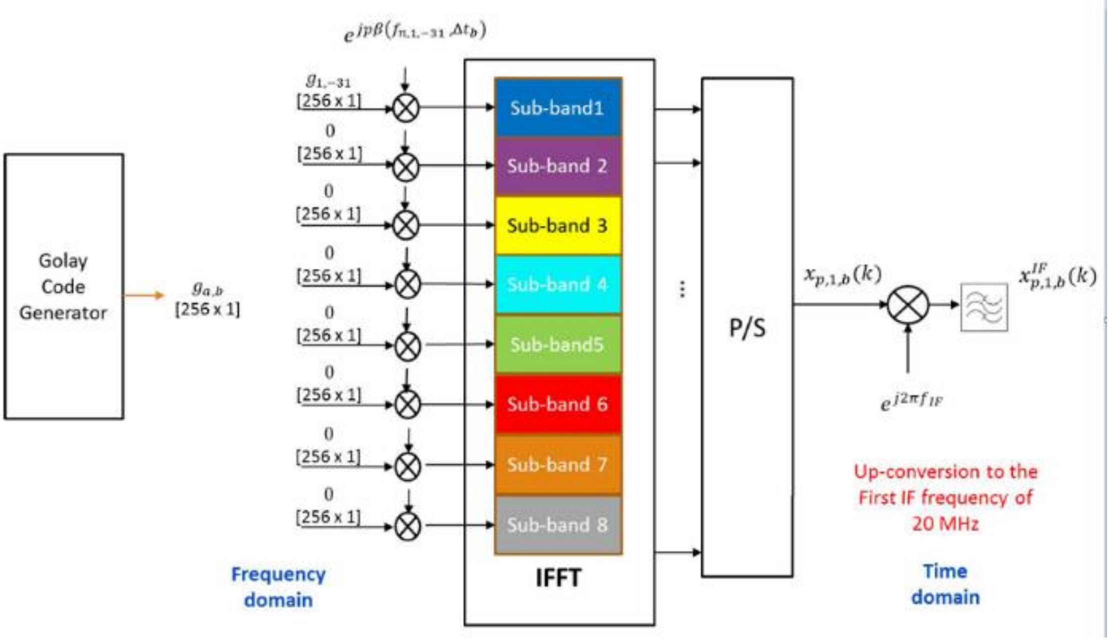


Fig. 2 Digital Waveform Generator for a specified beam *b*, on the p-th transmitter, which also involves digital up-conversion from baseband to the first IF frequency band [1].

III. HADAMARD AND GOLAY CODE GENERATORS

Golay codes are suitable candidates for OFDM MIMO radar because it has two important characteristics. The codes are orthogonal, and it has low PAPR value. Golay codes have a property that the summation of the autocorrelation between codes are equals to $2\delta(t)$ with zero sidelobes. The chosen codes will be assigned to the OFDM subcarriers, which are responsible for creating the θ_b beam. Then the length of the code will be equal to the number of subcarriers per beam N_{cb}.

In this paper, CC-OFDM MIMO radar waveforms for surveillance as specified in [1] is used. The number of subcarriers per beam,N_{cb}=256 subcarriers. It is related to the mission requirement for the low power mode, where range resolution of 60 meter per beam direction is needed. The desired codes must be orthogonal and, more importantly, they must give the lowest PAPR values. In the following, the two codes, Hadamard and Golay codes are compared with respect to their orthogonality and their PAPR performance.

A. Hadamard Codes

Hadamard codes have been known and used in communication systems because of their orthogonal properties [9-10]. Hadamard codes can be generated recursively using Walsh-Hadamard matrix generator:

$$H_N = \begin{bmatrix} \mathrm{H}_{N/2} & \mathrm{H}_{N/2} \\ \mathrm{H}_{N/2} & -\mathrm{H}_{N/2} \end{bmatrix} \tag{4}$$

with the initial value of $H_1 = 1$.

For the surveillance radar under consideration, generation of H_{256} will provide 256×256 matrix consisting of 256 codes with a code length of 256 chips. Out of 256 codes available 65 codes can be chosen to produce 65 orthogonal beams[1].

B. Golay Complementary Codes [3]

The Golay matrix generator is formulated by first initializing the Golay code matrix G for N = 1, with the value of $G_1 = \widetilde{\mathrm{G}}_1 = 1$, where $\widetilde{\mathrm{G}}_N$ is a commuted version of G_N, while N is the code length.

$$G_N = \begin{bmatrix} \mathrm{G}_{\frac{N}{2}} & \widetilde{\mathrm{G}}_{\frac{N}{2}} \\ \mathrm{G}_{\frac{N}{2}} & -\widetilde{\mathrm{G}}_{\frac{N}{2}} \end{bmatrix} \tag{5}$$

$$\widetilde{\mathrm{G}}_N = \begin{bmatrix} \mathrm{G}_{N/2} & -\widetilde{\mathrm{G}}_{N/2} \\ \mathrm{G}_{N/2} & \widetilde{\mathrm{G}}_{N/2} \end{bmatrix} \tag{6}$$

The codes are complementary when the sum of its autocorrelation is following:

$$\sum_{k=1}^{N} (x_k x_{k+i} + y_k y_{k+i}) = \begin{cases} 2N & i = 0 \\ 0 & i \neq 0 \end{cases} \tag{7}$$

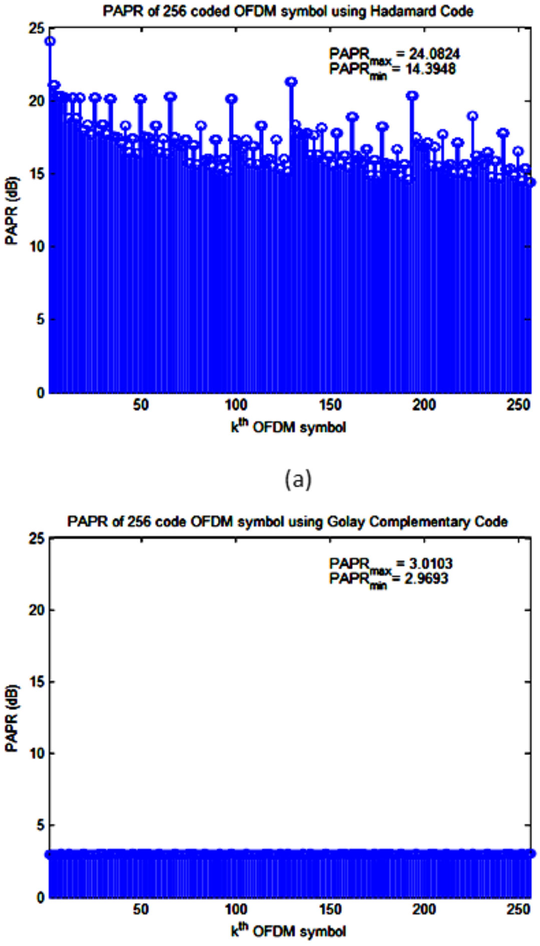
Taking the Fourier transform of both side of the equation results in the summation of the power spectrum of *X* and *Y*.

$$|X(f)|^2 + |Y(f)|^2 = 2N \tag{8}$$

Hence the spectrum is bounded by 2N, $|X(f)|^2 < 2N$. By assuming that the power of the sequence x is equal to 1, then the average power is N.

$$PAPR < \frac{2N}{N} \rightarrow PAPR < 3\text{ dB} \tag{9}$$

Subsequently, comparison is made between the resulting PAPR per-OFDM symbol coded using Hadamard code and Golay complementary code. It can be seen from Figure A.1 that the PAPR per symbol of Golay-coded OFDM is better than the Hadamard-coded one. Golay codes give PAPR around 3 dB while Hadamard gives PAPR in the range of 14-24 dB. This result agrees with Prasad’s analysis in [2].



(a)

(b)

Fig. 3 PAPR of each of 256 coded OFDM symbols with (a) Hadamard codes and (b) Golay codes.

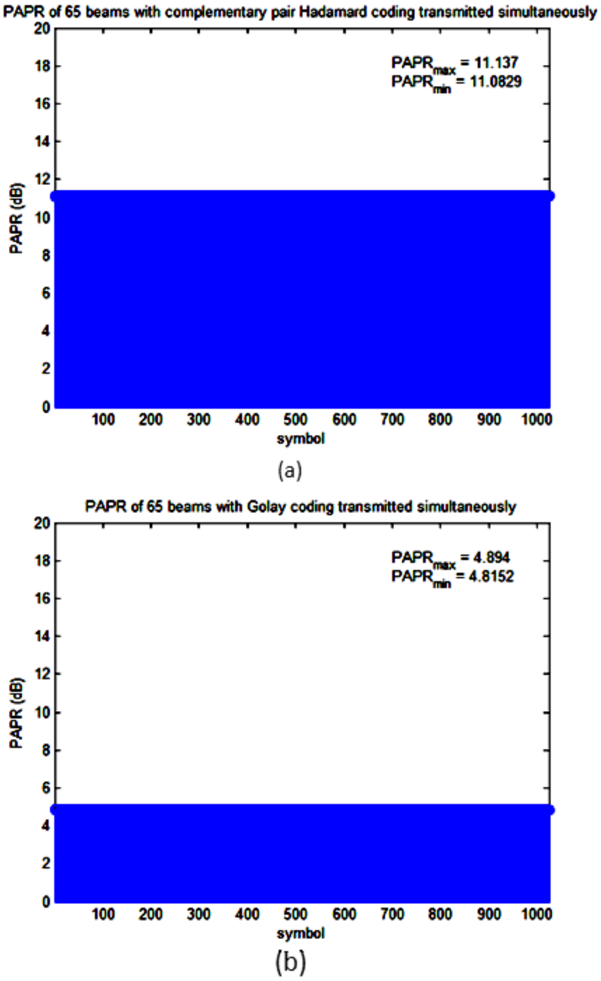
The PAPR of a single transmission of CC-OFDM waveform, which are coded with 256 variations of Hadamard and Golay codes are shown in Fig. 3a and Fig. 3b respectively. The PAPR value of OFDM waveform using Hadamard codes is relatively high in the range of 14.4 to 24 dB while for Golay coded CC-OFDM waveform the PAPR is around 3dB. From these results, for single CC-OFDM waveform transmission, Golay code superior to the Hadamard codes.

According to the radar specification in Table 1, 65 beams are going to be transmitted simultaneously using 8 sub-bands. Hence, there should be combinations for the 65 codes that will give the lowest PAPR values.

In this paper the PAPR of 1000 combinations of 65 codes in 8 sub bands are calculated and shown in Fig. 4. It can be seen that using Hadamard complementary code the PAPR is around 11 dB while with Golay complementary code the PAPR is around 4.9dB. Hence, the Golay codes still are superior to Hadamard complementary code. In order to attempt to reduce the PAPR value furthermore, the additional Selective Mapping (SLM) technique is applied to the CC-OFDM waveform.

C. *Selective Mapping (SLM) Technique [9-10]*

The SLM technique is implemented to the coded OFDM signals by the following procedures:



(a)

(b)

Fig. 4 PAPR of 65 beams CC-OFDM signals transmitted simultaneously, with: (a) Hadamard complementary codes, and (b) Golay complementary codes

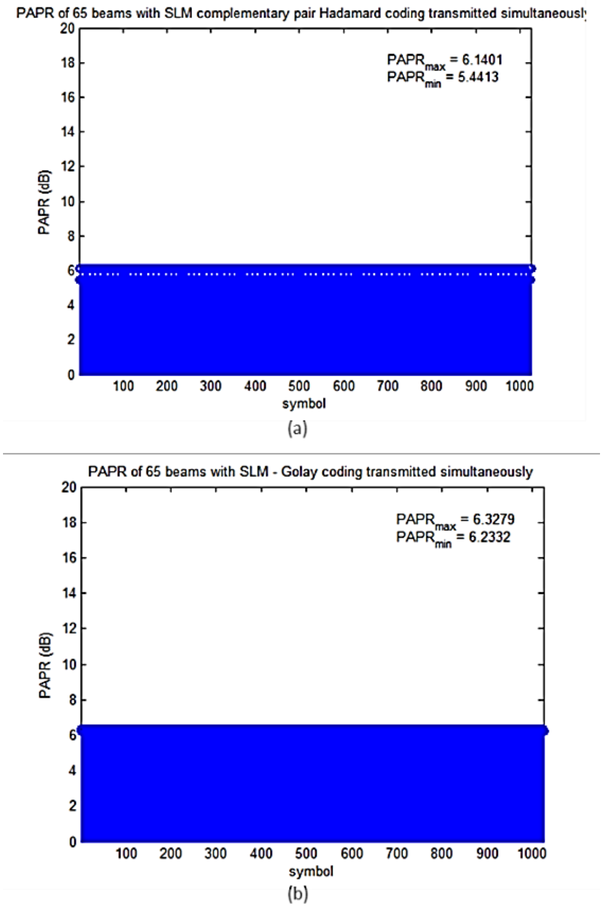


Fig. 5 PAPR of OFDM signals applying the SLM technique with (a) Hadamard codes and (b) Golay codes.

i. Shift the phase of the Hadamard/Golay codes before applying the IFFT to get the coded OFDM signal.
ii. Generate randomly the phase shift variation for each of the subcarrier. The phase shift has 4 possible values, namely {1, -1, *j*, –*j*}
iii. Multiply the Hadamard codes, which are basically a BPSK signal (1, -1), with the phase shift, so that they become like a 4-PSK signal.
iv. Apply IFFT to obtain the coded OFDM signal
v. Calculate the PAPR of the signal
vi. Repeat the step for all the possible phase shift combinations
vii. Compare the PAPR of the signal and find which combinations gives the lowest PAPR
viii. Save the phase combinations which give the lowest PAPR.

It can be seen in Fig. 5, that after implementing the additional SLM technique, the PAPR of the Hadamard coded OFDM signal is in the range of 5.4 to 6.1 dB, while with the Golay codes, the PAPR is around 6.2 to 6.3 dB.

Based on the comparison and analysis of the performance of the Hadamard and Golay codes for coding the subcarrier of the OFDM signal, regarding the PAPR value, Golay codes are superior to the Hadamard codes.

Besides the low PAPR values, the codes that are used for coding the beam must be orthogonal. Fig. 6 shows the correlation matrix of the 256 Golay and Hadamard codes. The two figures show that the correlation between the codes is zero, except when the code is correlated with itself. It means that both the codes are orthogonal.

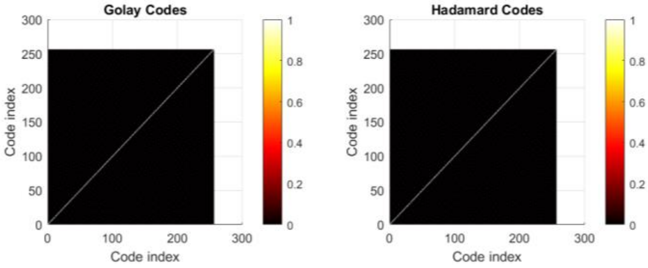


Fig. 6 Golay code and Hadamard correlation Matrix

IV. CONCLUSION

Based on the simulation and analysis that has been described in section III, it can be concluded that Golay code is the most suitable candidate for coding the CC-OFDM waveform compared to the Hadamard codes. These conclusions are drawn because based on the multibeam CC-OFDM radar waveform requirements, the beam signals must be orthogonal and have low PAPR values to avoid nonlinear problem at the power amplifier output. And Golay codes are proven to have fulfilled these requirements. Golay code is orthogonal and there are combinations of codes that could give the PAPR values around 4.8 - 4.9 dB. Hence it is a potential candidate for coding the multibeam CC-OFDM waveform.

REFERENCES

[1] D. Kuswidiastuti, L. P. Ligthart, and G. Hendrantoro, “A Transmitter Design for the Multi-beam CC-OFDM Azimuth Scanning MIMO Radar,” IEEE Access, vol. 10, pp. 53682–53702, May 2022, doi: 10.1109/ACCESS.2022.3175883.

[2] Zheng, Xunnan, et al. "PAPR reduction with compressive sensing for joint radar and communication system." 2022 7th International Conference on Signal and Image Processing (ICSIP). IEEE, 2022.

[3] Li, Wanlu, Zheng Xiang, and Peng Ren. "Waveform design for dual-function radar-communication system with golay block coding." IEEE Access 7 (2019): 184053-184062.

[4] Singh, S., & Kumar, A. (2016). Performance analysis of adaptive clipping technique for reduction of PAPR in Alamouti coded MIMO-OFDM systems. Procedia Computer Science, 93, 609–616.

[5] Nguyen, T. T., & Lampe, L. (2008). On partial transmit sequences for PAR reduction in OFDM systems. IEEE Transactions on Wireless Communication, 2, 746–755.

[6] Zhou, Y., & Jiang, T. (2009). A novel multi-point square mapping combined with PTS to reduce PAPR of

OFDM signals without side information. IEEE Transactions on Broadcasting, 55, 831–835.

[7] Yang, L., Soo, K. K., Li, S. Q., & Siu, Y. M. (2011). PAPR reduction using low complexity PTS to construct OFDM signals without side information. IEEE Transactions on Broadcasting, 57, 284–290.

[8] Elavarasan, P., & Nagarajan, G. (2015). Peak-power reduction using improved partial transmit sequence in orthogonal frequency division multiplexing systems. Computers & Electrical Engineering, 44, 80–90.

[9] Vittal, M. V. R., & Rama Naidu, K. (2017). A novel reduced complexity optimized PTS technique for PAPR reduction in wireless OFDM systems. Egyptian Informatics Journal, 18, 123–131.

[10] Xiaojing Huang, "Simple implementations of mutually orthogonal complementary sets of sequences," 2005 International Symposium on Intelligent Signal Processing and Communication Systems, Hong Kong, China, 2005, pp. 369-372, doi: 10.1109/ISPACS.2005.1595423.

[11] Sghaier, Mouna, Fatma Abdelkefi, and Mohamed Siala. "New SLM-Hadamard PAPR reduction scheme for blind detection of precoding sequence in OFDM systems." 2014 IEEE Wireless Communications and Networking Conference (WCNC). IEEE, 2014.

[12] Shankar, T., et al. "Hadamard based SLM using genetic algorithm for PAPR reduction in OFDM systems." 2017 Innovations in Power and Advanced Computing Technologies (i-PACT). IEEE, 2017.

[13] Prasad, Sanjana, and Ramesh Jayabalan. "PAPR reduction in OFDM systems using modified SLM with different phase sequences." Wireless Personal Communications 110.2 (2020): 913-929.

[14] R. Firat Tigrek and P. van Genderen, "A Golay code-based approach to reduction of the PAPR and its consequence for the data throughput," 2007 European Radar Conference, Munich, Germany, 2007, pp. 146-149, doi: 10.1109/EURAD.2007.4404958.